

Q.1 Find the transpose of each of the following matrices.

(I) $\begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$ (II) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (III) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Soln: (I) Let $A = \begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$, then $A' = [5 \ 7 \ -1]$, 1×3

(II) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(III) Let $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, then $A' = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Q.2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

then verify that (i) $(A+B)' = A' + B'$ (ii) $(A-B)' = A' - B'$

Soln. It is given that $A+B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$\Rightarrow A+B = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

$\Rightarrow (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$ ————— (I)

Also, $A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 3 \\ -5 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$ ————— (II)

From (I) and (II), we get $(A+B)' = A' + B'$ proved.

(ii) It is given that $A-B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$
 $= \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$

$$\Rightarrow (A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \quad \text{--- (I)}$$

$$\text{Also, } A'-B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

From (I) and II, we get $(A-B)' = A'-B'$ --- (II)

Q.3. If $A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then

Verify that (i) $(A+B)' = A'+B'$ (ii) $(A-B)' = A'-B'$

Soln: $\therefore A = (A')' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}' = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

(i) $A+B' = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$

$\therefore \text{L.H.S} = (A+B)' = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

$B' = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, also $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

$\therefore \text{R.H.S} = A'+B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

So, $(A+B)' = A'+B'$

(ii) R.H.S = $A'-B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

Also, $(A-B)' = \left(\begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \right)'$

$= \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

$\therefore (A-B)' = A'-B'$

Q.4. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)'$.

Soln. It is given that $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$

$$\Rightarrow A = (A')' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} \quad [\because (A')' = A]$$

$$\text{and } 2B = 2 \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$$

$$A+2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\Rightarrow (A+2B)' = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

Q.5. For the matrices A and B, verify that $(AB)' = B'A'$, where

$$\text{① } A = \begin{bmatrix} 1 & 4 \\ -4 & 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} \quad \text{② } A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$$

$$\text{Soln. ① } AB = \begin{bmatrix} 1 & 4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & 4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & 4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \quad \text{--- ③}$$

$$\text{Also, } B'A' = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}' \begin{bmatrix} 1 & 4 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

From ③ and ④, $(AB)' = B'A'$. — ⑤

Q.6. If ① $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then verify that $A'A = I$

$$\text{② } A = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}, \text{ then verify that } A'A = I$$

Soln. (i) $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$$\begin{aligned} \therefore A'A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha \cdot \cos \alpha + \sin \alpha \cdot \sin \alpha & \cos \alpha \cdot \sin \alpha + (-\sin \alpha) \cdot \cos \alpha \\ \sin \alpha \cdot \cos \alpha - \cos \alpha \cdot \sin \alpha & \sin \alpha \cdot \sin \alpha + \cos \alpha \cdot \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

(ii) $\therefore A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$\therefore A'A = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \cos \alpha \sin \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, $A'A = I$

Proved